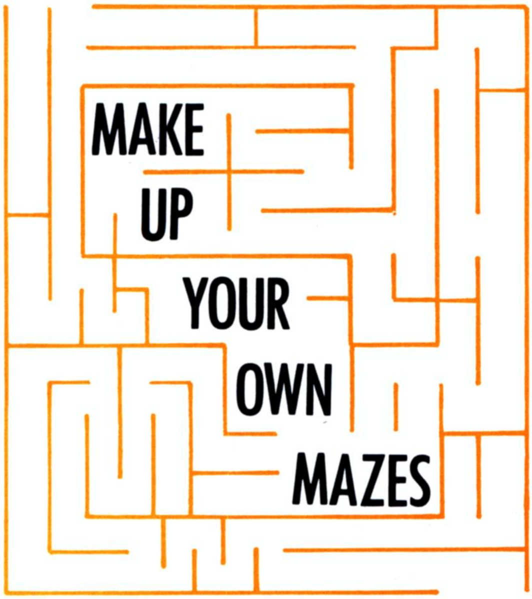
MAKE UP YOUR OWN MAZES

Author(s): EDWARD T. ORDMAN
Source: *The Arithmetic Teacher*, Vol. 22, No. 8 (DECEMBER 1975), pp. 650-652
Published by: National Council of Teachers of Mathematics
Stable URL: http://www.jstor.org/stable/41191235
Accessed: 28-02-2018 14:13 UTC



EDWARD T. ORDMAN

During five years as an assistant professor of mathematics at the University of Kentucky, Edward Ordman frequently taught prospective teachers and spoke in secondary schools. He is presently a Research Associate in the University of Kentucky's Bureau of Business and Economic Research.

A few years ago I showed a class of prospective elementary teachers a theorem that I had always assumed to be part of the folklore of topology. Apparently, however, it is little used today, either by teachers or by mathematical researchers. It was so well received by the class of prospective teachers that I then showed it to inservice teachers, and finally I visited a few elementary classes to try it on the pupils firsthand. It was regularly a success—in one instance pupils were so excited they showed it to friends during lunch and disrupted school for much of the afternoon. Even at the third-grade level, some pupils were able to follow the reasoning well enough to convince another classroom teacher (who had not seen the material in advance) of the truth of the theorem. At higher grade levels, the theorem continues to be appropriate for any pupils who have not yet had exposure to the "theorem-proof" arguments of the sort common in Euclidean geometry.

Look at the maze illustrated in figure 1—an ordinary maze with the start at the upper left and the finish at the lower right. When I was in elementary school, I was very enthusiastic about such mazes, my principal frustration being that I could not get enough of them. In particular, I was unable to make up mazes *to fool myself.* At best, a friend and I could make up and exchange mazes, but even this usually required the use of tracing paper or carbon paper to compose a maze without the answer being drawn in in advance.

There is, however, a simple process for making mazes. It is illustrated in figures 2 and 3. Begin by drawing a "box," open at the upper left and at the lower right, as in figure 2. Now add interior lines, one at a time, being careful to obey the following rule:

Each new line must touch exactly one *previously drawn line.*

Note that the line touched may be an inside or an outside line; the new line may end at the previously drawn line or may cross it; the new line may be curved, straight, or turn corners. The only requirement is that it touch at least one previously drawn line and no more than one previously drawn line. Continue to add new lines, one at a time, until the maze is one of any desired degree of complexity. (Young children may find this easier to do if they work on square-ruled or quadrille paper rather than on blank sheets.) (Figure 1 is the completed version of the maze shown being drawn in figures 2 and 3.)

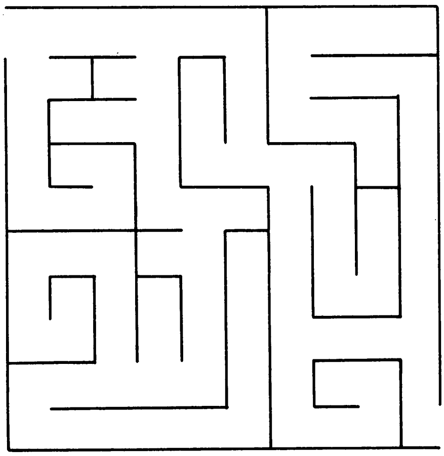


Fig. 1

Of course, a maze has little point unless there is a way to get from the start to the finish. An interesting and perhaps surprising fact can be expressed as follows:

A maze composed in the way described above will have exactly one solution.

The proof of this is easy to see. The original "box" drawing (fig. 2) has two pieces. At each stage, when a new line is drawn, the drawing continues to consist of two pieces—since the new line touches some earlier line, it does not start a third new piece, and since it touches only one previously drawn line, it cannot join the two pieces into one piece. Since the completed maze now must consist of two pieces also, there is exactly one path separating the two pieces—that path is the solution.

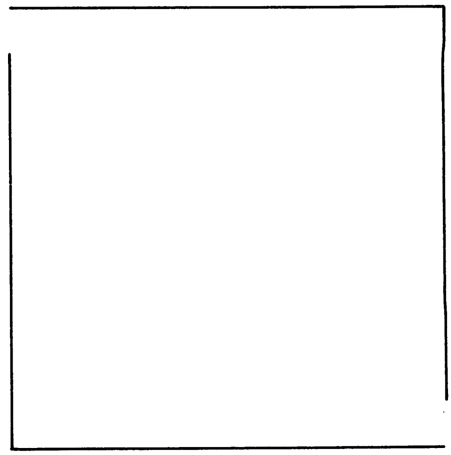


Fig. 2

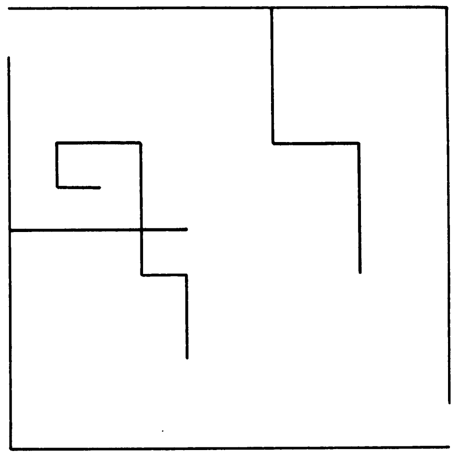


Fig. 3

(Note that if the completed maze were somehow one piece, as in figure 4, there would be no solution, and if there were more than two pieces as in figure 5, there would be more than one solution.)

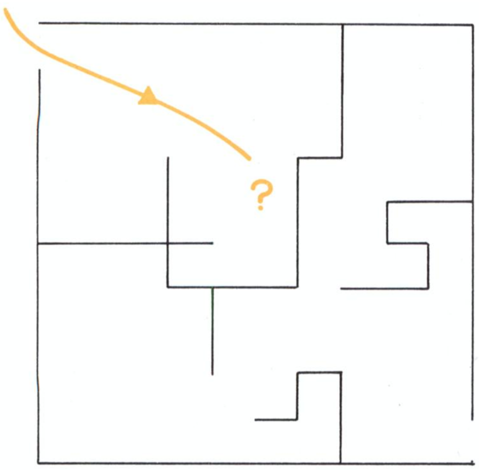


Fig. 4

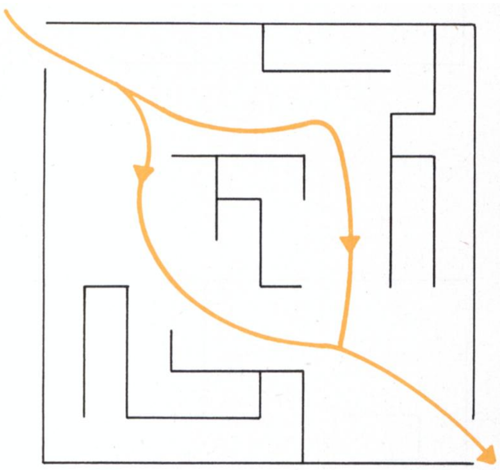


Fig. 5

As noted before, I have found the "proof" of this convincing and understandable to most fourth graders and some third graders. (Of course, one need not subject these pupils to the word *theorem*.) What is most intriguing to me is that the pupils immediately recognize this material as being mathematics—after all, it depends critically on the distinction between the numbers one, two, and three—but it is clearly *not* arithmetic, as there is no hint of addition or subtraction.

In the middle grades, pupils who discover that mazes are a legitimate subject for discussion may demand to know if there are "mathematical" or "right" ways to solve a maze. While not entirely satisfactory to some, the following technique will work for mazes of the type constructed above. Imagine yourself walking through the maze. When you enter, place your hand on the right-hand wall. Walk along, always keeping your hand on the right-hand wall. When you come to a corner, make the sharpest right turn possible. If you come to a dead end, turn around, turning *left* so that your right hand remains on the wall. You are now retracing a previous path, but the wall on your right is the opposite wall from the one that was on your right previously. In this way, you will eventually pass every part of the "right-hand" section of the maze; in particular, you will eventually pass through the "finish" at the bottom right of the maze. Brighter pupils in the upper grades may wish to further explore questions connected with this. Do they know other kinds of mazes (for instance, where the finish is a box in the center)? Can they compose a maze for which the keep-to-the-right rule will not work?