E 1657. *Proposed by Michael Gemignani, University of Notre Dame*

Let G be any group and A a subgroup of G. Let $x \in G$, $x \notin A$. We say x *augments* A if $A_x = A \cup \{x, x^{-1}\}$ is also a subgroup of G. Suppose x augments A. Show that A_x is cyclic of order 2, 3, or 4.

E 1658. *Proposed by D. L. Silverman, Beverly Hills, Calif.*

Points are selected at random on the circumference of a circle until they form the vertices of an inscribed polygon which encloses the center of the circle. Prove that the "expected polygon" is a pentagon.

E 1659. *Proposed by José Gallego-Diaz, Universidad del Zulia, Maracaibo, Venezuela*

A parabola has the property that the circumcircle of the triangle formed by three tangents to the curve passes through a fixed point (the focus). Does this property characterize the parabola?

E 1660. *Proposed by Seymour Kass, Illinois Institute of Technology*

Give an example of a strongly partially ordered set which has the property that every pair of unrelated elements has a sup and inf, while every pair of related elements has neither. (Strong partial order: antireflexive and transitive.)

SOLUTIONS

Horological Interchangeability

E 1571 [1963, 330]. *Proposed by J. L. Pietenpol, Columbia University*

How many times in a twelve hour period are the hands of a clock interchangeable (i.e., such that interchanging the positions of the hands yields a possible clock reading)?

Solution by D. C. B. Marsh, Colorado School of Mines. Measured in degrees clockwise from 12:00, at M minutes after H o'clock, the locations of the minute and hour hands are M and $5H+M/12$ respectively. For "interchangeability" we must satisfy

$$M' = 5H + M/12, \qquad 5H' + M'/12 = M,$$

with $H, H' \in \{1, 2, \cdots, 12\}$ and $0 \leqq M, M' < 60$. Solving

$$M = 60(H + 12H')/143, \qquad M' = 60(H' + 12H)/143,$$

we note that all pairs of H, H' yield solutions with only $H = H' = 1$ and $H = H' = 12$ giving the same reading—spaced 12 hours apart. Thus, during a twelve hour period there are $12^2 - 1 = 143$ times when the clock hands are interchangeable; of these, 11 are self-corresponding while the others form 66 dual pairs.

Also solved by J. C. Abad, R. H. Anglin, W. D. Anscher, K. F. Bailie, Frank Dapkus, Monte Dernham, P. J. Erdelsky, Robert Feinerman, Michael Goldberg, J. A. H. Hunter, A. R. Hyde, R. A. Jacobson, Emmett Keeler and Richard Zeckhauser (jointly), P. L. Kingston, E. F. Lang, Harry Langman, Coline Makepeace, Helen M. Marston, J. E. Morriello, P. N. Muller, J. B. Muskat, P. R. Nolan, E. T. Ordman, Stanton Philipp. S. J. Ryan, Jean-Pierre Sampson, D. L. Silverman, Guy Torchinelli, B. R. Toskey, Gary Venter, Andy Vince, Julius Vogel, W. C. Waterhouse, S. E. Weinstein, K. L. Yocom, A. R. Zingher, and the proposer.

Attention was called to Problem 61 in H. E. Dudeney's *Amusements in Mathematics* (Dover Publications, Inc., 1958) and to Exercise 19 of Chapter II in Harry Langman's *Play Mathematics* (Hafner Publishing Company, 1962).

Squares and Rectangles on a Chess Board

E 1572 [1963, 330]. *Proposed by Anders Bager, Hjørring, Denmark*

Enumerate the number of (1) squares, (2) rectangles, on an $n\times n$ "chess" board.

Solution by R. A. Jacobson, South Dakota State College. The number of squares of dimension $j\times j$ on an $n\times n$ chessboard is $(n+1-j)^2$. Hence the total number of squares is given by

$$\sum_{j=1}^{n}(n+1-j)^2 = n(n+1)(2n+1)/6.$$

The number of rectangles of dimension $j\times k$ on an $n\times n$ chess board is $(n+1-j)(n+1-k)$. It follows that the total number of rectangles is

$$\sum_{j=1}^{n}\sum_{k=1}^{n}(n+1-j)(n+1-k) = n^2(n+1)^2/4.$$

Also solved by J. C. Abad, R. G. Albert, J. A. Andrews and W. C. Waterhouse (jointly), R. H. Anglin, W. D. Anscher, Joseph Arkin, K. F. Bailie, B. W. Banks and J. R. Fall and Lawrence Lessner (jointly), M. J. Behr and William Roughead (jointly), W. G. Brady, Julian Braun, Robert Brooks, R. E. Brown, A. W. Brunson, D. I. A. Cohen, Charles Conlin, Frank Dapkus, J. F. Dillon, P. J. Erdelsky, Bruce Erickson, J. A. Faucher, S. T. Fisk, E. T. Frankel, C. M. Frye, Michael Gemignani, Michael Goldberg, Ralph Greenberg, R. E. Greenwood, J. C. Hennessey, K. D. Herr, J. A. H. Hunter, A. R. Hyde, Roman Kaluzniacki, Emmett Keeler and Richard Zeckhauser (jointly), C. L. Krueger, Joel Kugelmass, G. J. Kurowski and J. D. Watson (jointly), Harry Langman, H. R. Leifer, S. B. Leonard, Robert Maas, Coline Makepeace, Andrezj Makowski, C. F. Marion, D. C. B. Marsh, Helen M. Marston, R. A. Melter, Stephen Montague, J. E. Morriello, P. N. Muller, Amos Nannini, Sam Newman, L. S. Nicholson, E. T. Ordman, R. R. Perez, Stanton Philipp, E. M. Scheuer, R. R. Seeber, R. L. Syverson, Ronald Tannenwald, Rory Thompson, Dmitri Thoro, Guy Torchinelli, B. R. Toskey, Gary Venter, Andy Vince, Julius Vogel, S. E. Weinstein, Ron Wilder, and the proposer.

Editorial Note. Attention was called to J. A. H. Hunter and J. S. Madachy, *Mathematical Diversions* (D. Van Nostrand Col, Inc., 1963), p. 129, to H. E. Dudeney, *Amusements in Mathematics* (Dover Publications, Inc., 1958), Problem 347, to *Scripta Mathematica*, 1949, p. 100, to *School Science and Mathematics*, Problem 2859, and to this MONTHLY, Problem 1127 [1955, 183].

It is interesting that the number of squares on an $n\times n$ chess board is $\sum_{i=1}^{n} i^2$ and the number of rectangles is $\sum_{i=1}^{n} i^3$. The number of squares on an $m\times n$ chess board, $m\geqq n$, is

$$n(n+1)(3m+1-n)/6,$$

and the number of rectangles is

$$mn(m+1)(n+1)/4.$$