$$S(n,m) = \sum_{i_1=m}^{n} \binom{i_1 - 1}{m - 1} = \binom{n}{m}.$$

This last summation identity is well known; see, e.g., W. Feller, *An Introduction to Probability Theory and its Applications*, Vol. I (Second edition), 1957, p. 61.

III. *Solution by G. P. Steck, Sandia Laboratories, Albuquerque.* Let $k = n-m$ and consider any arrangement of m A's and k B's. Let the position in the sequence of the rth A from the right be i_r so that the rightmost A is in place i_1 and the leftmost A is in position i_m.

The required sum is the number of ways the A's and the B's can be arranged so that $1 \leqq i_m \leqq k+1$, $i_m + 1 \leqq i_{m-1} \leqq k+2, \cdots, i_2 + 1 \leqq i_1 \leqq m+k$. But these restrictions are automatically satisfied since $i_r > i_{r+1}$ and since the rth A from the right cannot be fewer than r places from the right hand end of the sequence. Consequently the required sum is the number of ways that m A's and k B's can be arranged in a sequence, which is $\binom{m+k}{m} = \binom{n}{m}$.

More general sums of the same type appear in ballot problems and in the two-sample problem of order statistics. In this latter context, I have showed that for given sequences of integers $a_1 \leqq a_2 \leqq \cdots \leqq a_m$ and $b_1 \leqq b_2 \leqq \cdots \leqq b_m$, $(i \leqq a_i \leqq b_i \leqq k+i)$, the number of ways m A's and k B's can be arranged so that $a_r \leqq i_{m-r+1} \leqq b_r$ $(r = 1,2,\cdots,m)$ is the determinant of the $m \times m$ matrix $M = (m_{ij})$ where

$$m_{ij} = \binom{b_i - a_j + j - i + 1}{j - i + 1}.$$

In the case at hand we have $a_i = i$ and $b_i = k + i$. See G. P. Steck, *The Smirnov two sample tests as rank tests*, Ann. Math. Stat. 40 (1969), 1449–1466; a simpler proof is given in S. G. Mohanty, *A short proof of Steck's result on two-sample Smirnov statistics*, Ann. Math. Stat. 42 (1971), 413–414.

Also solved by the proposer and $\binom{10}{2} + 1$ other contributors.

Editor's comment. John Ivie points out that the result can be obtained using generating functions and Pascal's triangle as in his article, *Multiple Fibonacci sums*, Fibonacci Quart. 7 (1969), 303–309. For a connection with lattice problems see C. A. Church and H. W. Gould, *Lattice point solution of the generalized problem of Terquem and an extension of Fibonacci numbers*, Fibonacci Quart. 5 (1967), 59–68. For a connection with Catalan numbers see Problem E 2054 [1969, 192].

Venn Again

E 2314 [1971, 904]. *Proposed by A. K. Austin, The University, Sheffield, England*

Prove or disprove that it is possible to find a convex polygon and three translations of it in the plane which form a Venn diagram for four sets (i.e., they form 16 connected regions and no three edges pass through the same point).

Solution by Heiko Harborth, Braunschweig, Germany. Any two congruent convex polygons that are related by a translation have at most two points of intersection, common arcs being considered as single points. If three such polygons meet in one point, then slight translations of one or two of them will form a small triangle in place of the point, increasing the number of regions by one. Thus we need only consider cases where the polygons intersect two by two in distinct points. A further permissible simplification now is the replacement of the convex polygons by circles. Then the Venn diagram for n such circles has $n(n-1)$ vertices and $2n(n-1)$ edges or arcs ($2n-2$ of each on each circle). By Euler's formula, the number of faces is given by

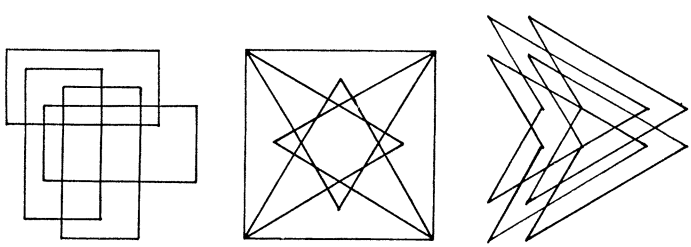
$$F = 2 + E - V = n^2 - n + 2 < 2^n$$

when $n \geqq 4$. Hence a Venn diagram for $n \geqq 4$ sets cannot be formed from any convex set and $n-1$ translations of it.

Also solved by Ken Brons, D. Ž Djoković, J. R. Kuttler, L. E. Mattics, E. T. Ordman, and F. G. Schmitt, Jr.

Editor's comment. Schmitt notes that the proof for circles appears in Yaglom & Yaglom, *Challenging Mathematical Problems with Elementary Solutions*, Vol. I, 1964, 103–104.

Five correspondents sent figures showing four congruent convex polygons (or ovals) forming a Venn diagram and related by translations and rotations. The figures below show such a diagram for rectangles (by G. A. Heuer, Concordia College) and for equilateral triangles (by the reviewer), each of which can be constructed using rotations only. The last figure (by the reviewer) shows four non-convex quadrilaterals related solely by translations. (The last two figures are not connected.)



Subdivisions of a Polygon

E 2315 [1971, 904]. *Proposed by Richard Stanley, Harvard University*

Let $f(n)$ be the number of ways an $(n+1)$-sided convex polygon can be divided into regions by diagonals not intersecting in the interior of the polygon. The trivial division, that is the division using no diagonals, is to be counted, so that $f(1) = 1$, $f(2) = 1, f(3) = 3, f(4) = 11$, etc. Find the generating function $F(x) = \sum f(n)x^n$, and find an asymptotic formula for $f(n)$.